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Superconducting ELF Magnetic Field Sensors for Submarine Communications

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The ferromagnetic-core solenoid used as a trailed antenna for communications reception by submarines has not demonstrated adequate sensitivity to satisfy the performance requirements of deeply submerged submarines receiving ELF transmissions from the planned SANGUINE communications system. It is possible, however, that a superconducting quantum interference device (SQUID) can be adapted to this role and satisfy all requirements of platform depth, speed and maneuverability.

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This report describes the current status of SQUIDs and presents some laboratory measurements that prove the potential of these devices for ELF communications reception. It also discusses the engineering development that must be pursued in the areas of (a) sensor and electronics dynamic range, (b) sensor orthogonality, and (c) refrigeration systems to make SQUID ELF sensors available as operational receiving antennas.

CONTENTS

SUPERCONDUCTING ELF MAGNETIC FIELD SENSORS FOR SUBMARINE COMMUNICATIONS

INTRODUCTION

Current submarine deployment plans require greatly improved communications between surface or airborne transmitting terminals and deeply submerged receivers. Most particularly, restrictions on receiver depth, speed, and maneuverability must be lessened. The attenuation of radio waves in seawater at very low frequencies (VLF) requires the receiving antenna to be relatively near the surface. Furthermore, the long trailing-wire antennas and ferromagnetic-core solenoids (which may be buoy-mounted) now in normal use at these frequencies hamper a submarine's operating flexibility.

The prospect of using the extremely low frequency (ELF) band in the near future for submarine communications provides a potential solution to some of these requirements that avoids the limitations of VLF communications systems. Principally, ELF waves penetrate so deeply into seawater that sensors need no longer be towed close to the surface. However, two serious problems remain:

- Trailing-wire (distributed-electrode) and ferromagnetic-core solenoid sensors are long and consequently unwieldy, and thus limit maneuverability.
- Omnidirectionality requires either an impractical deployment of widely separated electrode pairs (one pair can be trailed, but the other pair must be deployed perpendicular to the direction of motion) or development of a relatively noise-free solenoid.

As will be indicated in the following section, present solenoid sensors cannot satisfy the noise requirements for adequate performance in likely operating areas at the planned ELF transmitting capacity and the required message-delivery rate. This report proposes the development of a magnetic-field (H-field) sensor that can satisfy the performance requirements and provide fully omnidirectional receiving capability without the need for an electric field (E-field) sensor at all. This proposed sensor will employ a Superconducting Quantum Interference Device (SQUID) and will permit ambient noise level performance to be achieved in the bandwidth required for ELF communications.

The third section of this report describes a typical SQUID, discusses laboratory measurements performed recently at the Naval Research Laboratory that confirm SQUID sensitivity in the ELF range, and sets forth the three major hurdles to effective employment of a SQUID in an operational receiving system: Achievement of adequate dynamic range, achievement of adequate orthogonality among three planar sensors, and development of improved means of refrigeration.

Note: Manuscript submitted January 17, 1974.

REQUIRED SENSOR PERFORMANCE

In the case of the proposed SANGUINE ELF communications system, a submarine ELF H-field sensor must have a sensitivity equal to or greater than -200 dB relative to 1 $V/m\cdot\sqrt{Hz}$, in a bandwidth 100 Hz wide centered at a frequency of 80 Hz. This sensitivity permits the requirements listed above to be met in the ordinary sea operating areas during the noisiest season. The bandwidth requirement of 100 Hz permits wideband noise processing to be used to reduce the impact of the highly non-Gaussian noise in the ELF band on the communications signal.

Current efforts to achieve omnidirectionality in submarine reception are centered on efforts to supplement the E-field trailing-wire antenna with an H-field antenna of the ferromagnetic-core solenoid type. Such an antenna is limited by several types of noise, including motion-induced noise of two kinds (end effect and distortion noise), magnetostriction noise, and thermal noise of the core winding. The best demonstrated sensitivity for an H-field antenna to date is -185 dB relative to 1 $V/m\cdot\sqrt{Hz}$, or 15 dB worse than the required performance. This may be near-maximum performance for this type of device; it is possible that satisfactory performance cannot be achieved with the ferromagnetic-core solenoid. A new approach seems warranted.

As indicated in the third section SQUID sensors have been shown at the Naval Research Laboratory to be capable of detecting about 2×10^{-15} $T/\sqrt{Hz}$ (in seawater, a magnetic field of 10^{-14} $T/\sqrt{Hz}$ is equivalent to the required sensitivity of -200 dB relative to 1 $V/m\cdot\sqrt{Hz}$) in the ELF band above 100 Hz. Measurements at lower frequencies were not possible because of power frequency interference in the vicinity of 60 Hz, but there is no reason to doubt that this same sensitivity is available over the entire planned communications bandwidth. SQUIDs are subject to motion-related noise, but unlike the magnetostriction and cable-distortion noise to which a long solenoid is subject, which are due to internal motion, the major noise problem in SQUIDs is controlling the relatively rigid rotations of the sensors in the earth's magnetic field. If this motion-related noise can be eliminated, the SQUID sensor can provide a self-contained omnidirectional receiving capability equal to the requirements listed above.

The motion-related noise to be accommodated consists of the effect of sensor rotation in the earth's magnetic field of $5 \times 10^{-5} T$. Because the earth's field is uniform on the physical scale of the typical SQUID, however, a set of orthogonal sensors can be used to sum the vector components continuously and thus permit the earth's field component to be removable as a DC component. The problems are (a) providing enough dynamic range in the parts of the system ahead of the vector summing stage to prevent the SQUID electronics from being saturated, and (b) achieving orthogonality among the sensors. The total dynamic range requirement is about 194 dB. Using techniques developed for magnetic airborne detection, physical rotation of the sensor can be constrained to less than 10^{-3}rad, thus lowering the required sensor and electronics dynamic range to 134 dB. Presentday SQUID electronics have a dynamic range of 140 dB. The 134-dB electronics dynamic range and 100-Hz bandwidth can be achieved by using 24-bit digital electronics and a 9.6-kHz sampling rate. A means for achieving effective orthogonality among the sensors will now be discussed.

APPLICATION OF SUPERCONDUCTING QUANTUM INTERFERENCE DEVICES

Measured SQUID Characteristics

The required sensitivity of 10^{-14} $T/\sqrt{Hz}$ should be achievable with a presently available SQUID system combined with a superconducting flux transformer. The typical thin-film SQUID sensor, as shown in Fig. 1, is a cylindrical film of superconducting material. in the form of a ring 2 mm in diameter deposited on an insulating substratum and has a submicron constriction at one point along the circumference.* This ring is coupled inductively to a resonant circuit tuned to 30 MHz, and the radiofrequency voltage across the coupled structure is measured as a function of the external magnetic field. In this configuration the SQUID sensitivity is nearly constant over the frequency range of 0 to 30 kHz. The impedance of the ring is a function of the external magnetic field once a characteristic drive current has been exceeded, and it is this quality that makes a SQUID useful as a magnetic field sensor. The response of the superconducting ring to the applied flux is periodic, with the period being one quantum of flux defined as $h/2e = 2 \times 10^{-11}$ $T{\cdot}cm^2$ (h is Planck's constant, and e is the electronic charge). Typically, the signal-to-noise ratio is such that one part in 10^3 of a flux quantum can be resolved in a 1-Hz bandwidth. For a ring 2 mm in diameter this yields a sensitivity of 6×10^{-13} $T/\sqrt{Hz}$.

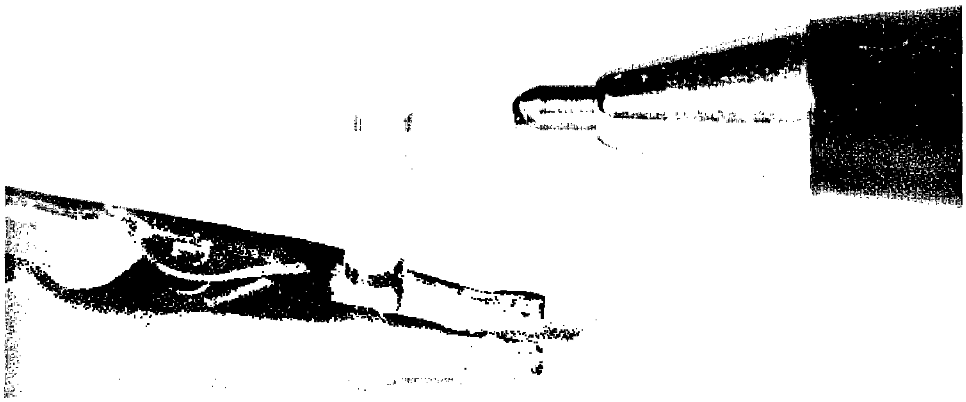

Fig. 1 — A typical SQUID sensor consisting of a superconducting ring, 2 mm in diameter and about 1 mm high, evaporated on a quartz cylinder. The ring has a bow-tie constriction at one point along the circumference. On the right is the tip of a ballpoint pen for reference.

*See Proc. IEEE, January 1973.

The sensitivity of the SQUID can be substantially improved by using a superconducting flux transformer to couple the SQUID to the external field. A flux transformer consists of two coils made of a continuous, simply connected wire or ribbon of superconducting material. One coil is coupled inductively to the SQUID cylinder, and the other coil is exposed to the external flux. If the external field is changed, the properties of the superconducting state demand that a current flows in the flux transformer to keep the enclosed flux constant, and this current causes a corresponding change in field to be produced at the SQUID. A simple calculation taking into account the relative areas of the two coils, the inductance of the coils, and the coupling of the SQUID coil to the SQUID indicates that field amplication can be obtained in this fashion.†

Two such flux transformers were fabricated at NRL and coupled to a SQUID that had a peak-to-peak instrumental noise corresponding to 6×10^{-13} $T/\sqrt{Hz}$ for signals from DC to 30 kHz. One flux transformer provided a field amplication of 7 and had a peak-to-peak noise at DC and in the frequency range from 100 Hz to 10 kHz such that a signal corresponding to $(9 \pm 5) \times 10^{-14}$ $T/\sqrt{Hz}$ could be readily detected. Data were not taken in the vicinity of 60 Hz because the ambient noise near 60 Hz was about 10^{-5} T, peak to peak. The attenuation provided by two superconducting shields reduced the noise at the flux transformer to about one flux quantum, which is a factor of 10^3 larger than the signal that was detected at frequencies away from 60 Hz. The other flux transformer gave a field amplication of 300 and consisted of a single turn of 5-cm-wide lead foil 15 cm in diameter (Fig. 2). The measured sensitivity in this case was such that the peak-to-peak noise was $(2 \pm 2) \times 10^{-15}$ $T/\sqrt{Hz}$ measured at DC and from 100 Hz to 30 kHz. At frequencies near 60 Hz, the environment introduced additional noise above that intrinsic to the system. Additional shielding was provided in this case by a large metal Dewar flask. One important result of the experiments was that despite the 60-Hz

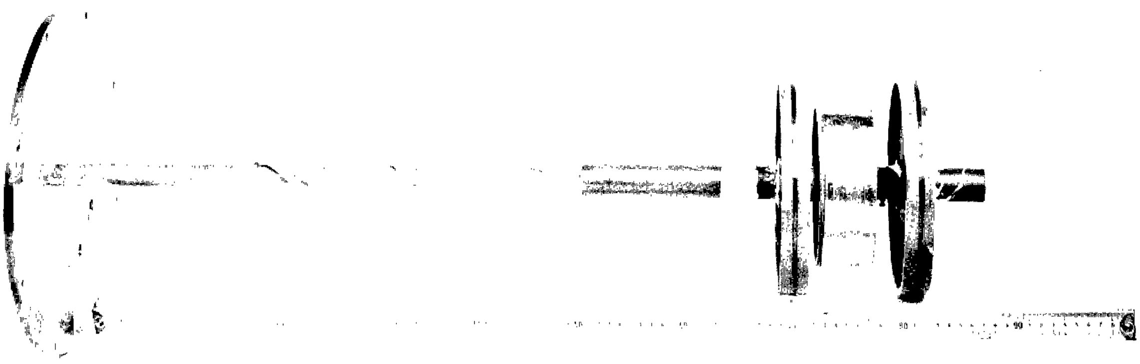

Fig. 2 — The 5-cm-long lead-foil flux transformer primary 15 cm in diameter, which when coupled to the basic SQUID as described in the text has a 2×10^{-15} $T/\sqrt{Hz}$ sensitivity. The two larger copper coils are the 20-cm-in-diameter Helmholtz pair used to provide the calibration field. The SQUID itself is mounted between the two Helmholtz coils on the axis of the flux transformer. The long rod and large circular plate are for attachment to a Dewar flask.

†J. E. Zimmerman, J. Appl. Phys. 42, 4483 (1971).

noise, as great as several flux quanta in the second experiment described above, it was still possible to resolve signals of 10^{-3} flux quantum at frequencies removed from the frequency of the noise.

Stability Requirements

One of the most serious problems in using the $10^{-14} T/\sqrt{\mathrm{Hz}}$ sensitivity of a SQUID is the unwanted signal generated by the device moving in the earth's magnetic field. Since the field of the earth is 5×10^{-5} T, it is necessary to cancel the within-band effects of motion in this field to 2 parts in 10^{10}. The requirements are substantially relaxed for noise components outside the operating bandwidth.

If the sensors are completely free to rotate, each sensor would have to be capable of tracking a field from 10^{-14} to 5×10^{-5} T, resulting in a required 194 dB dynamic range. In addition, if the orthogonality of two sensors were in error by θ, then the change in the vector sum of the outputs of these sensors for a 90° rotation of the sensors would be equal to $H_e\ \theta$, where H_e is the earth's field. Thus, if no signal greater than 10^{-14} T due to the rotation of misoriented sensors is to be allowed, then θ must be 2×10^{-10}rad or less; equivalently, the sensors must be orthogonal to 2 parts in 10^{10}. These requirements are very stringent but can be greatly relaxed if one of the sensors is in the direction of the earth's field.

A simple calculation (see appendix) indicates that if one sensor's alignment is maintained along the earth's field H_e to within an angle α, and if the worst misalignment in orthogonality of sensors is θ, then the largest motion-induced signal is (for small α, θ) approximately $H_e \alpha\theta$. A reasonable goal for α might be 10^{-3}rad, because this degree of stabilization has been achieved in airborne magnetic anomaly-detection systems. With this degree of alignment, the dynamic range required would be 134 dB, and dynamic ranges of 140 dB have already been achieved electronically in the frequency range of interest. However, this circumstance also requires effective orthogonality of the sensors to 2 parts in 10^7. If a flux transformer diameter of 10 cm is assumed, this degree of orthogonality corresponds to alignment of the plane of the flux transformer to 200 Å. Clearly, a subsidiary alignment procedure must be incorporated.

It can be shown (see appendix) that the variation with θ of the signal from two misoriented sensors is proportional to $(\sin\alpha \cos\alpha)\theta$. If the best alignment possible results in an orthogonality of about 1 part in 10^4, then a feedback scheme could be used to electronically cancel the motion-induced signal associated with the residual misalignment.

A conceptually simple method of employing such a compensation scheme would begin by placing the mechanically rigid three-sensor system in a uniform calibration field, with one sensor (say the x sensor) aligned in the direction of the field. Small departures of the other two sensor axes from orthogonality with the x sensor could be compensated by combining their outputs with just enough of that from the x sensor to null them. The sensors then could be rotated by 90° so that one of the others (say the y sensor) is aligned with the field, and the x sensor is nulled. Then the remaining z sensor output could be combined with just enough of the y sensor output to yield a null. Other, more sophisticated methods also could be employed. For example, two fluxgate sensors with sensitivities of 10^{-6} T and orthogonal to 1 part in 10^4 would, if their outputs were electronically multiplied, give a signal varying as $\sin\alpha \cos\alpha$. A part of this signal could be fed back into the SQUID coils to cancel the signal due to the SQUID's motion in the field. Two fluxgates would be required for each SQUID not aligned along the earth's field. Balancing the system would be done empirically by adjusting the amount of feedback from each pair of fluxgates until the output from the SQUIDs was no longer sensitive to small motions. If α could be kept below 10^{-3}rad, then $\sin\alpha \cos\alpha$ would be approximately equal to α, and one fluxgate per misaligned sensor would be all that was needed. Although cumbersome, this procedure should be workable.

Frequency components of the motion-induced noise outside the signal band also need to be considered. The product of this noise amplitude (measured in flux quanta) and the noise frequency must be less than the rate at which the SQUID electronics can "count" flux quanta, or its slewing rate. The presently achievable slewing rate is about 10^{-6} quanta per second. For a SQUID plus flux transformer with a sensitivity of 10^{-14} $T/\sqrt{\mathrm{Hz}}$ a quantum corresponds to 10^{-11} T. So, for example, noise at 10 Hz could be tolerated up to 10^{-6} T. For frequencies in the motion-induced noise that are outside the signal band, if one of the orthogonal sensors is oriented along the earth's field to 10^{-3}rad, then orthogonality of the sensors would not be required to keep the error signal due to rotation smaller than 10^{-14} T, but only to ensure that the vector-summed signal amplitude remains constant for a signal of arbitrary polarization.

Cryogenic Environment and Refrigeration

The remaining major problem associated with the use of SQUIDs is their cryogenic environment. Present-day SQUIDs require a liquid-helium bath to provide an operating temperature in the vicinity of 4.2 K. If (assuming the worst case, in which hull-mounting might be prevented by excessive noise at that location) the SQUID must be located in a separate vehicle towed a considerable distance behind the submarine, the refrigeration must either be aboard the towed vehicle or the vehicle must contain a sufficient reservoir of liquid helium to provide adequate deployment time. Three alternatives are possible: A vehicle containing a Dewar flask, a vehicle containing a Gifford-McMann closed-cycle refrigerator with its compressor aboard the submarine, and a vehicle containing a Joule-Thomson refrigerator using three working gases and compressors or gas cylinders aboard the submarine.

In the simplest case, a large portion of the vehicle would be a vacuum insulated container or Dewar flask to hold the liquid helium required to provide the necessary refrigeration. The cable to the submarine would be relatively simple and would consist of a tension member and electrical leads. However, if using a Dewar flask of reasonable size, the vehicle would have to be brought aboard the submarine for helium replenishment at intervals of less than 30 days. This fairly short deployment time may be unsatisfactory for operational situations but would not be a drawback in initial tests.

In the second alternative the towed vehicle would contain a Gifford-McMann closed-cycle refrigerator, with the compressor aboard the submarine. The main advantage of this system is that the vehicle could remain deployed indefinitely. The disadvantages are that the refrigerator has a moving piston with a frequency of motion of about 1.3 Hz, and this might cause motion to the SQUID mounting (it could, however, be made with non-magnetic parts so as not to give rise to any magnetic signature). The high-pressure helium gas and the low-pressure return would have to run in cables from the submarine to the vehicle, and these cables would be quite heavy because they would have to handle a large volume of gas. The refrigerator could be made quite small (about 0.03 m^3), and several built for space applications have demonstrated 8000 hr. between required maintenance.

In the third alternative, a Joule-Thomson refrigerator using three working gases would be located on the vehicle and the compressors or compressed-gas cylinders would be on the submarine. This type of refrigerator would use nitrogen, hydrogen, and helium as the working gases and would have no moving parts; it would use expansion valves for refrigeration. While this method requires nitrogen and hydrogen at a pressure of about 100 atmospheres and helium at 30 to 40 atmospheres on the high pressure sides, it does not use a large volume of these gases and the high-pressure lines could consist of tubing of extremely small inside diameter. These tubes could be concentric with the low-pressure return lines. This method requires six passages for gas flow, but the cabling would weigh less than in the Gifford-McMann case. The drawbacks to this system are that it requires three compressors which in their present form are large and cumbersome, and that the refrigerator in its present form is not very reliable. Further development work on both compressors and refrigerator would be required. A commercially available research refrigerator of this type is illustrated in Fig. 3.

To summarize, the first alternative, employing a Dewar flask in a towed vehicle, could be implemented with existing cryogenics, provided the time between refillings could be made adequately long and the servicing time acceptably short. The noise characteristics of the Gifford-McMann refrigerator would have to be studied and a reliable compressor developed before it became a viable option. This method also has the disadvantage of requiring heavy cabling. The Joule-Thomson refrigerator also requires compressor development and lighter but more complicated gas coupling between submarine and towed vehicle. Each alternative requires some development beyond what is available now.

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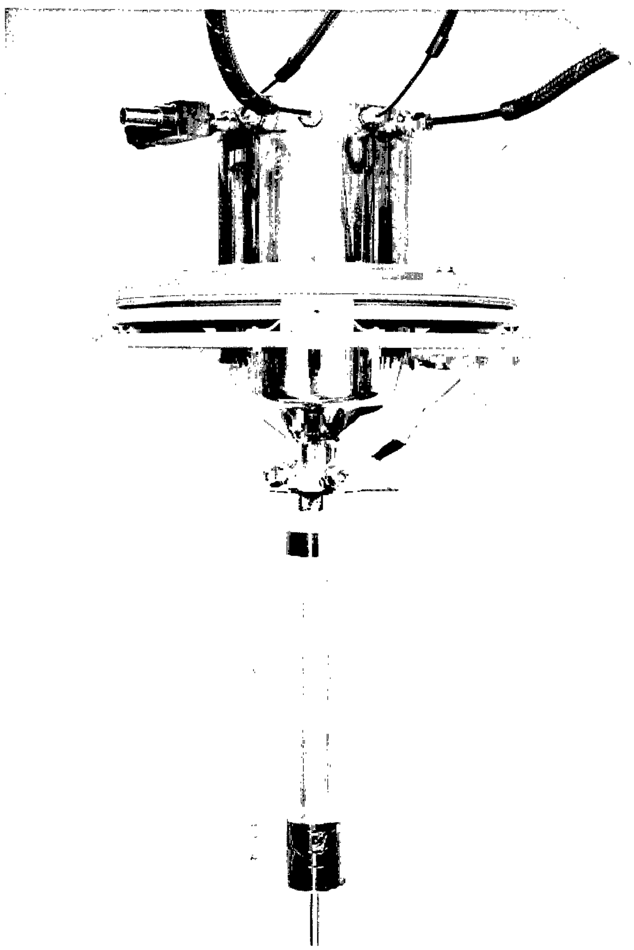

Fig. 3 — A commercially available Joule-Thomson refrigerator (the cylindrical object adjacent to the ruler), which liquifies both bottled hydrogen and helium without the use of any moving parts. The nitrogen in this version is supplied as a liquid; with an additional expansion valve, the nitrogen could also be liquified. This would not make the device significantly larger or more complex.

Appendix A

ANGULAR DEPENDENCE OF THE MOTION-INDUCED NOISE OF A THREE-COMPONENT MAGNETOMETER ON DEVIATIONS FROM ORTHOGONALITY

It is desired to make a magnetic field sensor, using SQUIDs, that is insensitive to motion in the earth's field. One technique would be to form the vector sum of the outputs of three precisely orthogonal sensors. Motion of a system of misaligned sensors will cause noise to be generated. This noise will be calculated in the case where one sensor is along the x axis, one sensor is misaligned with the y axis by an angle θ in the xy plane, and one sensor is misaligned with the z axis by an angle φ in the xz plane. This is not the most general case but it has all the types of error terms which would enter the general calculation. Figure A1 illustrates the angles involved in the calculation.

From Fig. A1 it may be seen that the field measured by the three sensors would be

$$H^2_{\text{meas}} = [H_e \cos\alpha \sin\beta]^2 + [H_e \cos\alpha \cos(\beta+\theta)]^2 + [H_e \cos\gamma \cos(\delta+\varphi)]^2 \qquad \text{(A1)}$$

which can be simplified using the following identities:

$$\cos\gamma \sin\delta = \cos\alpha \sin\beta$$

$$\cos\gamma \cos\delta = \sin\alpha.$$

If θ and $\varphi << 1$ then $\sin\theta \approx \theta$, $\sin\varphi \approx \varphi$, $\cos\theta = 1 - \theta^2/2$, $\cos\varphi = 1 - \varphi^2/2$, and after extracting the square root

$$H_{\text{meas}} = H_e \left\{ 1 + \frac{1}{2}\cos^2\alpha[\theta^2(\sin^2\beta - \cos^2\beta) - 2\theta \sin\beta \cos\beta] + \frac{1}{2}\varphi^2[\cos^2\alpha \sin^2\beta - \sin^2\alpha] - \varphi \cos\alpha \sin\beta \sin\alpha \right\} + \ldots \qquad \text{(A2)}$$

If the derivatives are taken with respect to α and β and multiplyed by $\Delta\alpha$ and $\Delta\beta$, respectively, the amplitude of the noise generated by small rotations α and β is

$$\Delta H^{\alpha}_{\text{meas}} = \frac{dH}{d\alpha}\Delta\alpha = H_e\{-\cos\alpha \sin\alpha[\theta^2(\sin^2\beta - \cos^2\beta) - 2\theta \sin\beta \cos\beta] - \varphi^2 \sin\alpha \cos\alpha[\sin^2\beta + 1] + \varphi \sin\beta[\sin^2\alpha - \cos^2\alpha]\}\,\Delta\alpha \qquad \text{(A3)}$$

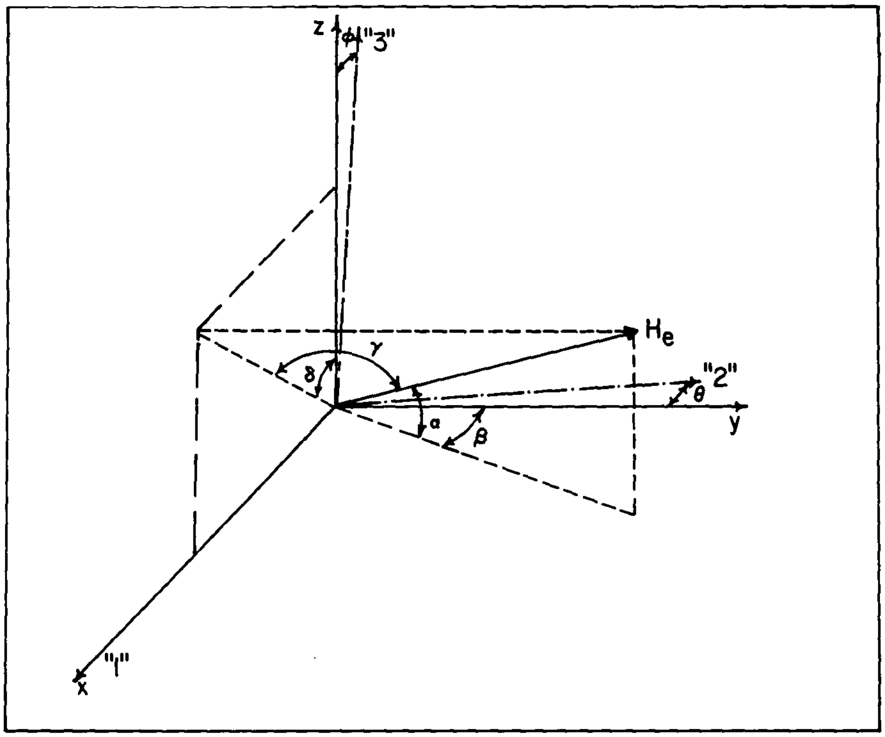

Fig. A1 — Misalignment of sensors. H_e is the earth's field. It makes an angle α with the xy plane, and its projection on the xy plane makes an angle β with the y axis. H_e makes an angle γ with the xz plane, and the projection of H_e on the xz plane makes an angle of δ with the z axis. The three orthogonal sensors are 1, 2, and 3. Sensor 1 is aligned with x, but 2 makes and angle θ with the y axis in the xy plane, and 3 makes an angle φ with the z axis in the xz plane.

and

$$\Delta H^{\beta}_{\text{meas}} = \frac{dH}{d\beta}\Delta\beta = H_e\left\{\frac{1}{2}\cos^2\alpha[4\theta^2 \sin\beta\cos\beta - 2\theta(\cos^2\beta-\sin^2\beta)] + \varphi^2\cos^2\alpha\sin\beta\cos\beta - \varphi\cos\alpha\sin\alpha\cos\beta\right\}\Delta\beta. \tag{A4}$$

For H_e aligned along the x axis, $\beta = 90°$, and $\alpha = 0$. Then from eqs. (A3) and (A4)

$$\Delta H^{\alpha}_{\text{meas}} = -H_e\,\varphi\Delta\alpha$$

$$\Delta H^{\beta}_{\text{meas}} = +H_e\,\theta\Delta\beta.$$

Thus for one sensor aligned along H_e the error is of first order in the angles describing nonorthogonality of the sensors, as well as first order in the deviation in alignment of the x sensor with H_e. There are combinations of angles α and β for which one or the other of $\Delta H^{\alpha}_{\text{meas}}$ and $\Delta H^{\beta}_{\text{meas}}$ varies quadratically with the angles describing nonorthogonality, but there is no combination for which both terms are quadratic.

The form of the deviation of the measured field H_{meas} from the actual field H_e will now be considered. From Eq. (A2),

$$H_{\text{meas}} - H_e = H_e\left\{\frac{1}{2}\cos^2\alpha[\theta^2(\sin^2\beta-\cos^2\beta) - 2\theta\sin\beta\cos\beta] + \frac{1}{2}\varphi^2[\cos^2\alpha\sin^2\beta-\sin^2\alpha] - \varphi\cos\alpha\sin\alpha\sin\beta\right\}.$$

For $\beta = 90°$

$$H_{\text{meas}} - H_e = H_e\left\{\frac{1}{2}\theta^2\cos^2\alpha + \frac{1}{2}\varphi^2\cos^2\alpha - \varphi\sin\alpha\cos\alpha\right\};$$

so to first order in the angles which measure nonorthogonality, but to higher order in α

$$H_{\text{meas}} - H_e = -H_e\varphi\sin\alpha\cos\alpha,$$

Similarly, for $\alpha = 0$,

$$H_{\text{meas}} - H_e = H_e\left\{\frac{1}{2}\theta^2(\sin^2\beta-\cos^2\beta) - \theta\sin\beta\cos\beta + \frac{1}{2}\varphi^2[\sin^2\beta]\right\}.$$

Thus, to first order in the angles which measure nonorthogonality, and higher order in β,

$$H_{\text{meas}} - H_e = H_e\theta\sin\beta\cos\beta.$$